

$$(1) A = \left\{ x \in \mathbb{N} \mid \frac{21}{2x+1} \in \mathbb{N} \right\}$$

$$\frac{21}{2x+1} \in \mathbb{N} \text{ deci } 2x+1 \in \mathcal{D}_{21} \text{ (numeri naturali)}$$

$$2x+1 \in \{1, 3, 7, 21\} \mid -1$$

$$2x \in \{0, 2, 6, 20\} \mid :2$$

$$x \in \{0, 1, 3, 10\}$$

$$\Rightarrow A = \{0, 1, 3, 10\}$$

$$(2) A = \left\{ x \in \mathbb{N} \mid \frac{18}{2x-1} \in \mathbb{Z} \right\}$$

$$\frac{18}{2x-1} \in \mathbb{Z} \text{ deci } 2x-1 \in \mathcal{D}_{18} \text{ (întregi)}$$

$$2x-1 \in \{-18, -9, -6, -3, -2, -1, 1, 2, 3, 6, 9, 18\} \mid +1$$

$$2x \in \{-17, -8, -5, -2, -1, 0, 2, 3, 4, 7, 10, 19\} \mid :2$$

$$x \in \left\{ -\frac{17}{2}, -4, -\frac{5}{2}, -1, -\frac{1}{2}, 0, 1, \frac{3}{2}, 2, \frac{7}{2}, 5, \frac{19}{2} \right\}$$

$$\text{deci } x \in \mathbb{N} \Rightarrow x \in \{0, 1, 2, 5\}$$

$$\Rightarrow A = \{0, 1, 2, 5\}$$

$$c) A = \left\{ x \in \mathbb{N} \mid \frac{7x+9}{2x-1} \in \mathbb{N} \right\}$$

$$\frac{7x+9}{2x-1} \in \mathbb{N} \text{ allora } 2x-1 \mid 7x+9 \quad | \cdot 2$$

$$\text{da } 2x-1 \mid 2x-1 \quad | \cdot 7.$$

$$2x-1 \mid 14x+18.$$

$$2x-1 \mid 14x-7 \quad (-)$$

$$2x-1 \mid (14x+18) - (14x-7)$$

$$2x-1 \mid 14x+18-14x+7.$$

$$2x-1 \mid 25.$$

$$2x-1 \in \{25\} \Rightarrow 2x-1 \in \{1, 5, 25\} \quad | +1$$

$$2x \in \{2, 6, 26\} \quad | :2$$

$$x \in \{1, 3, 13\}.$$

$$A = \{1, 3, 13\}.$$

$$d) A = \left\{ x \in \mathbb{Z} \mid \frac{5x+13}{2x+1} \in \mathbb{Z} \right\}$$

$$\frac{5x+13}{2x+1} \in \mathbb{Z} \text{ allora } 2x+1 \mid 5x+13 \quad | \cdot 2$$

$$2x+1 \mid 2x+1 \quad | \cdot 5.$$

$$2x+1 \mid 10x+26$$

$$2x+1 \mid 10x+5 \quad (-)$$

$$2x+1 \mid (10x+26) - (10x+5)$$

$$2x+1 \mid 10x+26-10x-5$$

(2)

$$2x+1 \mid 21$$

$$2x+1 \in \{-21, -7, -3, -1, 1, 3, 7, 21\} \mid -1$$

$$2x \in \{-22, -8, -4, -2, 0, 2, 6, 20\} \mid :2$$

$$x \in \{-11, -4, -2, -1, 0, 1, 3, 10\}$$

$$A = \{-11, -4, -2, -1, 0, 1, 3, 10\}$$

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63. Determinați perechile de numere naturale (a, b) pentru care.

e) $(a, b) = 12$ și $a \cdot b = 2160$.

Deci. $(a, b) = 12 \Rightarrow \begin{cases} a = 12 \cdot x \\ b = 12 \cdot y \end{cases}$ cu condiția $(x, y) = 1$ (prime între ele)

$$\Rightarrow 12 \cdot x \cdot 12 \cdot y = 2160$$

$$144xy = 2160$$

$$xy = 2160 : 144$$

$$x \cdot y = 15$$

I $\begin{matrix} x=3 \\ y=5 \end{matrix} \Rightarrow \begin{matrix} a=12 \cdot 3=36 \\ b=12 \cdot 5=60 \end{matrix} \quad (36, 60)$

II $\begin{matrix} x=1 \\ y=15 \end{matrix} \Rightarrow \begin{matrix} a=12 \cdot 1=12 \\ b=12 \cdot 15=180 \end{matrix} \quad (12, 180)$

III $\begin{matrix} x=5 \\ y=3 \end{matrix} \Rightarrow \begin{matrix} a=12 \cdot 5=60 \\ b=12 \cdot 3=36 \end{matrix} \quad (60, 36)$

$$\text{IV} \quad \begin{array}{l} x = 15 \\ y = 1 \end{array} \quad \begin{array}{l} a = 12 \cdot 15 = 180 \\ b = 12 \cdot 1 = 12 \end{array} \quad (180, 12)$$

$$b) (a; b) = 15 \quad a \cdot b = 3150.$$

$$\begin{array}{l} a = 15 \cdot x \\ b = 15 \cdot y \end{array} \quad \text{au condition } (x, y) = 1$$

$$15 \cdot x \cdot 15 \cdot y = 3150$$

$$225xy = 3150$$

$$xy = 3150 : 225$$

$$xy = 14 \quad \text{avec } (x, y) = 1 \Rightarrow$$

$$\Rightarrow \begin{array}{l} x = 2 \Rightarrow a = 15 \cdot 2 = 30 \\ y = 7 \Rightarrow b = 15 \cdot 7 = 105 \end{array} \quad (30, 105)$$

$$\text{II} \quad \begin{array}{l} x = 7 \Rightarrow a = 15 \cdot 7 = 105 \\ y = 2 \Rightarrow b = 15 \cdot 2 = 30 \end{array} \quad (105, 30)$$

$$\text{III} \quad \begin{array}{l} x = 1 \Rightarrow a = 15 \cdot 1 = 15 \\ y = 14 \Rightarrow b = 15 \cdot 14 = 210 \end{array} \quad (15, 210)$$

$$\text{IV} \quad \begin{array}{l} x = 14 \Rightarrow a = 15 \cdot 14 = 210 \\ y = 1 \Rightarrow b = 15 \cdot 1 = 15 \end{array} \quad (210, 15)$$

$$c) (a, b) = 15 \quad a + b = 120.$$

$$\text{Donc } (a, b) = 15 \Rightarrow \begin{array}{l} a = 15 \cdot x \\ b = 15 \cdot y \end{array} \quad \text{au condition } (x, y) = 1.$$

$$15x + 15y = 120. \quad x + y = 8$$

$$15(x + y) = 120. : 15$$

$$x + y = 8, (x, y) = 1.$$

$$\text{I } \begin{array}{l} x = 7 \Rightarrow a = 15 \cdot 7 = 105 \\ y = 1 \Rightarrow b = 15 \cdot 1 = 15 \end{array} \quad (105, 15)$$

$$\text{II } \begin{array}{l} x = 1 \Rightarrow a = 15 \cdot 1 = 15 \\ y = 7 \Rightarrow b = 15 \cdot 7 = 105 \end{array} \quad (15, 105)$$

$$\text{III } \begin{array}{l} x = 3 \Rightarrow a = 15 \cdot 3 = 45 \\ y = 5 \Rightarrow b = 15 \cdot 5 = 75 \end{array} \quad (45, 75)$$

$$\text{IV } \begin{array}{l} x = 5 \Rightarrow a = 15 \cdot 5 = 75 \\ y = 3 \Rightarrow b = 15 \cdot 3 = 45 \end{array} \quad (75, 45)$$

$$d) \quad \begin{array}{l} (a; b) = 18 \\ [a; b] = 270. \end{array}$$

AVEM FORMULA
 $(a; b) \cdot [a; b] = a \cdot b.$

$$(a, b) \cdot [a; b] = a \cdot b.$$

$$18 \cdot 270 = a \cdot b$$

$$a \cdot b = 4860$$

So, $(a; b) = 18 \Rightarrow \begin{array}{l} a = 18 \cdot x \\ b = 18 \cdot y. \end{array}$ cu condiția $(x; y) = 1.$

$$18x \cdot 18y = 4860$$

$$324xy = 4860$$

$$xy = 4860 : 324$$

$$xy = 15$$

$$x \cdot y = 15 \quad m(x, y) = 1.$$

$$\text{I} \quad \begin{array}{l} x=3 \\ y=5 \end{array} \Rightarrow \begin{array}{l} a = 18 \cdot 3 = 54 \\ b = 18 \cdot 5 = 90 \end{array} \quad (54, 90)$$

$$\text{II} \quad \begin{array}{l} x=5 \\ y=3 \end{array} \Rightarrow \begin{array}{l} a = 18 \cdot 5 = 90 \\ b = 18 \cdot 3 = 54 \end{array} \quad (90, 54)$$

$$\text{III} \quad \begin{array}{l} x=1 \\ y=15 \end{array} \Rightarrow \begin{array}{l} a = 18 \cdot 1 = 18 \\ b = 18 \cdot 15 = 270 \end{array} \quad (18, 270)$$

$$\text{IV} \quad \begin{array}{l} x=15 \\ y=1 \end{array} \Rightarrow \begin{array}{l} a = 18 \cdot 15 = 270 \\ b = 18 \cdot 1 = 18 \end{array} \quad (270, 18)$$

$\frac{26}{63}$ Determinați valorile naturale ale lui x pentru care:

e) $x+1 \mid x+6$

$$\begin{array}{r} x+1 \mid x+6 \\ x+1 \mid x+1 \quad (-) \end{array}$$

$$x+1 \mid x+6 - (x+1)$$

$$x+1 \mid 5$$

$$\begin{array}{l} x+1 \in \mathcal{D}_5 \quad x+1 \in \{1, 5\} \quad | -1 \\ x \in \{0, 4\} \end{array}$$

$$b) \quad x+2 \mid 2x+29$$

$$x+2 \mid x+2 \quad | \cdot 2$$

$$x+2 \mid 2x+29$$

$$\underline{x+2 \mid 2x+4 \quad (-)}$$

$$x+2 \mid 2x+29 - (2x+4)$$

$$x+2 \mid 2x+29 - 2x-4$$

$$x+2 \mid 25$$

$$x+2 \in \mathcal{D}_{25}$$

$$x+2 \in \{1, 5, 25\} \quad | -2$$

$$x \in \{-1, 3, 23\} \text{ dar } x \in \mathbb{N}$$

$$\Rightarrow x \in \{3, 23\}.$$

Temă : c) $2x-1 \mid 4x+19$

d) $2x+1 \mid 7x+23$

e) $2x+5 \mid 7x+29$

f) $3x+4 \mid 5x+17.$

$\frac{27}{63}$ Arătați că fracțiile următoare sunt ireducibile, oricare ar fi $m \in \mathbb{N}$.

a) $\frac{3m+11}{4m+15}$.

fracție este ireducibilă deci:

$$(3m+11, 4m+15) = 1.$$

presupunem că există un divizor d.
astfel încât $d \mid 3m+11$ 1.4

$$d \mid 4m+15 \quad 1.3$$

$$d \mid 12m+44$$

$$\begin{array}{r} d \mid 12m+45 \quad (-) \\ \hline d \mid (12m+45) - (12m+44) \end{array}$$

$$d \mid 12m+45-12m-44$$

$$d \mid 1 \Rightarrow d = 1.$$

Temă: $\frac{6m+13}{4m+9}$, $\frac{2m+5}{3m+8}$, $\frac{4m+9}{5m+11}$.